MWD TOOLFACE-INDEPENDENT ERROR TERMS

This section details how the MWD toolface-dependent terms can be replaced with toolface independent terms. The equations for the toolface-independent terms are based on work by Torkildsen and Bang in SPE 63275 (Reference CUR: 547) and further developed Andy Brooks.

The toolface-dependent terms remaining in ISCWSA MWD error model Revision 2 are:

ABX, *ABY*, *ABIX*, *ABIY*, *MBX*, *MBY*, *MBIX*, *MBIY*
ASX, *ASY*, *ASIX*, *ASIY*, *MSX*, *MSY*, *MSIX*, *MSIY*

All of these are assumed to propagate systematically.

The mathematics for developing equivalent toolface-independent weighting functions are given by Torkildsen and Bang in SPE 63275 (Reference CUR: 547). Their Appendix E uses toolface-dependent weights of the form:

$$a \cdot \sin^2 \alpha + b \cdot \sin \alpha \cdot \cos \alpha + c \cdot \cos^2 \alpha + d \cdot \sin \alpha + e \cdot \cos \alpha + f$$

for which it derives five equivalent toolface-independent weights

$$\left(\frac{a+c}{2} + f\right), \left(\frac{a-c}{\sqrt{8}}\right), \left(\frac{b}{\sqrt{8}}\right), \left(\frac{d}{\sqrt{2}}\right), \text{and} \left(\frac{e}{\sqrt{2}}\right)$$

For systematic toolface-dependent terms, the first of these terms always propagates systematically, while the other four may be either Systematic (S) or Random (R) depending on whether the drillstring is modeled as sliding or rotating (shown as S/R below). Multiple terms which propagate randomly may be Root-Sum-Squared (RSS) together into a single term.

The toolface-dependent terms in the ISCWSA MWD error model can be divided into two categories; biases (first order in $\sin\alpha$ and $\cos\alpha$, containing only elements *d* and *e*) and scale factor errors (second order in $\sin^2\alpha$, $\sin\alpha \cdot \cos\alpha$ and $\cos^2\alpha$, containing only elements *a*, *b*, and *c*). The terms always occur in symmetrical pairs corresponding to an *x* axis and *y* axis sensor, each of which leads to an identical toolface-independent weighting, and therefore we can combine each pair by multiplying the single-term weights by $\sqrt{2}$. The toolface-independent weights representing pairs of biases are therefore simply *d* and *e*, while those representing pairs of scale factor errors are:

$$\left(\frac{a+c}{\sqrt{2}}\right), \left(\frac{a-c}{2}\right), \text{and} \left(\frac{b}{2}\right)$$

where the coefficients are evaluated for either the *x* or *y* sensor, but not both.

INTEQ error models have implemented the systematically propagating (sliding mode) form of these toolface-independent bias terms, avoiding the additional complexities associated with the scale factor terms by including them in lumped bias terms. The gyro error model published in SPE 90408 (Reference CUR:599 and 709) included both random and systematic versions of the terms where appropriate; however it simplified the accelerometer terms by omitting their azimuth components, and it lumped together two or three axes for the *xy* or *xyz* gyro scale factor errors by RSS and sometimes by additional approximation.

Each pair of toolface-dependent terms generates either two (for biases) or three (for scale factor errors) equivalent toolface-independent terms. A complete set of toolface-independent weighting functions is given in Table 1 below. Each weighting function is given in the form of a vector with three components corresponding to the depth, inclination, and azimuth weighting functions.

The suffix "S" indicates that the term propagates systematically, while "S/R" means the term may be systematic for sliding mode or random for rotating mode.

The weighting function name as been based on the term name with 'TI' (for Toolface Independent) appended following by the number of the equivalent term. The standardised Term Code for specifying the magnitude of the error term is the weighting function name appended with S or R as appropriate e.g. accelerometer bias equivalent term 2 with a systematic propagation would be ABXY-TI2S

Table 1 MWD Model Toolface-Independent Terms

Term	Wt.Fn	Depth	Incl.	Azimuth	S/R
Accelerometer Biases					
ABX/ABY	ABXY-TI1	0	$-\frac{\cos I}{G}$	$\frac{\tan\theta\cdot\cos I\cdot\sin A_m}{G}$	S/R
	ABXY-TI2*	0	0	$\frac{(\cot I-\tan\theta\cdot\cos A_m)}{G}$	S/R
ABIX/ABIY	ABIXY-TI1	0	$-\frac{\cos I}{G}$	$\frac{\cos^2 I\cdot\sin A_m\cdot(\tan\theta\cdot\cos I+\sin I\cdot\cos A_m)}{G\cdot(1-\sin^2 I\cdot\sin^2 A_m)}$	S/R
	ABIXY-TI2 *	0	0	$-\frac{(\tan\theta\cdot\cos A_m-\cot I)}{G\cdot(1-\sin^2 I\cdot\sin^2 A_m)}$	S/R
Magnetometer Biases					
MBX/MBY	MBXY-TI1	0	0	$-\frac{\cos I\cdot\sin A_m}{B\cdot\cos\theta}$	S/R
	MBXY-TI2	0	0	$\frac{\cos A_m}{B\cdot\cos\theta}$	S/R
MBIX/MBIY	MBIXY-TI1	0	0	$-\frac{\cos I\cdot\sin A_m}{B\cdot\cos\theta\cdot(1-\sin^2 I\cdot\sin^2 A_m)}$	S/R
	MBIXY-TI2	0	0	$\frac{\cos A_m}{B\cdot\cos\theta\cdot(1-\sin^2 I\cdot\sin^2 A_m)}$	S/R
Accelerometer Scale Factor Errors:					
ASX/ASY	ASXY-TI1	0	$\frac{\sin I\cdot\cos I}{\sqrt{2}}$	$-\frac{\tan\theta\cdot\sin I\cdot\cos I\cdot\sin A_m}{\sqrt{2}}$	S
	ASXY-TI2	0	$\frac{\sin I\cdot\cos I}{2}$	$-\frac{\tan\theta\cdot\sin I\cdot\cos I\cdot\sin A_m}{2}$	S/R
	ASXY-TI3	0	0	$\frac{(\tan\theta\cdot\sin I\cdot\cos A_m-\cos I)}{2}$	S/R
ASIX/ASIY	ASIXY-TI1	0	$\frac{\sin I\cdot\cos I}{\sqrt{2}}$	$-\frac{\sin I\cdot\cos^2 I\cdot\sin A_m\cdot(\tan\theta\cdot\cos I+\sin I\cdot\cos A_m)}{\sqrt{2}\cdot(1-\sin^2 I\cdot\sin^2 A_m)}$	S
	ASIXY-TI2	0	$\frac{\sin I\cdot\cos I}{2}$	$-\frac{\sin I\cdot\cos^2 I\cdot\sin A_m\cdot(\tan\theta\cdot\cos I+\sin I\cdot\cos A_m)}{2\cdot(1-\sin^2 I\cdot\sin^2 A_m)}$	S/R
	ASIXY-TI3	0	0	$\frac{(\tan\theta\cdot\sin I\cdot\cos A_m-\cos I)}{2\cdot(1-\sin^2 I\cdot\sin^2 A_m)}$	S/R
Magnetometer Scale Factor Errors:					
MSX/MSY	MSXY-TI1	0	0	$\frac{\sin I\cdot\sin A_m\cdot(\tan\theta\cdot\cos I+\sin I\cdot\cos A_m)}{\sqrt{2}}$	S
	MSXY-TI2	0	0	$\frac{\sin A_m\cdot(\tan\theta\cdot\sin I\cdot\cos I-\cos^2 I\cdot\cos A_m-\cos A_m)}{2}$	S/R
	MSXY-TI3	0	0	$\frac{(\cos I\cdot\cos^2 A_m-\cos I\cdot\sin^2 A_m-\tan\theta\cdot\sin I\cdot\cos A_m)}{2}$	S/R

Term	Wt.Fn	Depth	Incl.	Azimuth	S/R
MSIX/MSIY	MSIXY-TI1	0	0	$\frac{\sin I \cdot \sin A_m \cdot (\tan\theta \cdot \cos I + \sin I \cdot \cos A_m)}{\sqrt{2} \cdot (1 - \sin^2 I \cdot \sin^2 A_m)}$	S
	MSIXY-TI2	0	0	$\frac{\sin A_m \cdot (\tan\theta \cdot \sin I \cdot \cos I - \cos^2 I \cdot \cos A_m - \cos A_m)}{2 \cdot (1 - \sin^2 I \cdot \sin^2 A_m)}$	S/R
	MSIXY-TI3	0	0	$\frac{(\cos I \cdot \cos^2 A_m - \cos I \cdot \sin^2 A_m - \tan\theta \cdot \sin I \cdot \cos A_m)}{2 \cdot (1 - \sin^2 I \cdot \sin^2 A_m)}$	S/R

Where:

- I = Inclination (deg)
- A_m = Magnetic Azimuth (deg)
- Θ = Magnetic Dip Angle (deg)
- B = Magnetic Field Strength (nT)
- G = Gravity Field Strength (ms^{-2})

* Note: The equation for terms ABXY-TI2 and ABIXY-TI2 are singular in vertical holes and need to have the additional term to calculate the error vectors in vertical holes where the weighting function is singular as in SPE 67616 Table A1

The equivalent weighting function for error vectors in vertical hole where the weighting function is singular is:

ABXY-TI2 and ABIXY-TI2

$$\begin{bmatrix} \frac{-\sin Az}{G} \\ \frac{\cos Az}{G} \\ 0 \end{bmatrix}$$